Digital Blockchain Trapezium-Spherical Two-Dimensional Circle Ledger

1. Abstract

This report presents a cryptographically chained, block-structured data ledger documenting a strict sequence of geometric derivations for an expanding observable universe model. Utilizing a fixed dataset of comoving radii, discrete temporal expansions, and calculated bounding lengths, the framework maps three-dimensional spatial coordinates into a two-dimensional topological format. The derivation chain enforces mathematical continuity through cryptographic hashing while preserving verbatim dataset inputs to expose dimensional transgressions. The ledger strictly governs spatial approximations, ultimately evaluating the mathematical limits of applying discrete polyhedral surface assumptions to continuous curved manifolds.

2. Methods

Cosmological Geometry and the Flat-Space Linear Approximation

The measurement of cosmological spaces relies extensively on the accurate parameterization of continuous manifolds. The observable universe, typically modeled as an isotropic and homogeneous manifold under the strictures of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, presents a continuously expanding volume. Within this regime, geometric derivations—such as calculating the comoving radius, the surface area of spherical boundaries, and the extruded volume of cosmic expansion—require rigid mathematical frameworks to prevent structural paradoxes in downstream mapping.

It is necessary to explicitly state that the methodology utilized within this ledger relies fundamentally on a flat-space, linear-H_0 approximation, which operates as a strict limitation of the model. Standard cosmological frameworks, such as the \LambdaCDM model, define the expansion of the universe through a time-varying Hubble parameter, H(t), which evolves based on the shifting density parameters of radiation, dark matter, and dark energy within the universe. The scale factor of the universe, a(t), accelerates dynamically, particularly in the dark-energy-dominated epoch. By enforcing a flat-space (\Omega_k = 0) and a strictly linear approximation of the Hubble constant fixed at H_0 = 0.0716 Gyr^{-1}, the spatial extrapolations executed over vast temporal spans—specifically the +1750 Myr leap to derive R_{future}—inherently oversimplify the true astrophysical expansion rate.

Under this flat-space, linear-H_0 limitation, the spatial geometry is treated as purely Euclidean at any given cross-section of cosmic time. Consequently, the calculations for spatial circumferences (C = 2\pi R), annular surface areas, and extruded volumes abandon the complex integrals of relativistic cosmology in favor of linear scalar adjustments. The future comoving radius is extrapolated via a purely linear kinematic function derived from the present baseline radius R_{today} = 46.5 Gly. While this linear, non-accelerating approximation deliberately sacrifices cosmological exactitude regarding dark energy's expansive pressure, it provides a highly stable, computationally linear baseline necessary for validating the discrete

digital ledger's architectural integrity and topographical mapping limits.

Transposition of Three-Dimensional Topographies to Two-Dimensional Ledgers

The core mechanical challenge of the digital data ledger is the projection of a dynamic, three-dimensional expanding manifold onto a restricted, two-dimensional discrete computational framework. The mapping of this continuous expansion into a two-dimensional ledger format necessitates the projection of spherical, spatial coordinates onto a lower-dimensional topological structure. This manifests mathematically as a trapezium-spherical transformation, wherein the curved latitudinal boundaries of the expanding cosmological sphere are flattened into planar computational elements representing distinct temporal epochs.

To maintain structural coherence in this reduced dimensionality, the framework establishes bounding parameters using the maximal cross-sectional circumference—the equatorial great circle. At the baseline epoch, this is defined by $C_{inner} = 292.168$ Gly, corresponding exactly to the linear Euclidean projection of the initial comoving radius ($2\pi \times 46.5$ Gly). As the system projects +1750 Myr forward, the newly calculated radius $R_{future} = 52.325$ Gly establishes a terminal boundary circumference, $C_{outer} = 328.744$ Gly. The intervening expansion is mapped discretely as an annular geometric plane.

The transformation enforces a strict calculation of this two-dimensional intervening space, defined mathematically as the annulus area. By utilizing the exact formula $\pi(R_{future}^2 - R_{today}^2)$, the ledger calculates an area of 1808.3 Gly^2. Because the physical universe expands outward radially in three dimensions, projecting this expansion as a flat two-dimensional annulus fundamentally ignores the spatial curvature and the z-axis depth of the spherical shell. To reconcile this topological reduction, the ledger incorporates an extruded volume calculation. This operates by integrating the annulus area over an orthogonal depth vector derived from the radial differential, specifically calculated as $dR/3$. This formulation yields an extruded volume of 3511.0 Gly^3, serving as a pseudo-three-dimensional reconciliation matrix within the otherwise planar ledger constraints.

Cryptographic Provenance and the Block-Structured Framework

The sequential calculations mapping these geometric transitions are bound within an immutable digital ledger utilizing a cryptographic blockchain architecture. In computational topography, particularly when integrating discrete temporal expansions such as the +25 Myr intermediary step ($R_{new} = 46.583$ Gly), the validity of a derived geometric property is absolutely dependent on the preceding dimensional states. The ledger mechanism isolates each mathematical derivation into an independent data block, indexing the operation chronologically.

To prevent the retroactive distortion of spatial measurements or the silent correction of irreconcilable topological mappings, the ledger employs a rigid SHA-256 cryptographic hashing algorithm. Every individual block within the sequence concatenates its discrete index, step label, input variables, mathematical formula, resulting output, and manually flagged boundary anomalies into a standardized JSON string. The algorithmic digestion of this string produces a unique 256-bit hash.

Critically, each block must ingest and record the full hash of the immediately preceding block (prev_block_hash), weaving an unbroken mathematical chain linking the final boundary declarations back to the genesis parameters of R_{today}. If a theoretical maximum is

exceeded—such as Length B exhibiting an anomalous circumference of +36.06 Gly beyond the theoretical maximum of C_{new}—the ledger is structurally forbidden from altering the fundamental dataset to "smooth" the error. The input values must be executed exactly as codified, with inconsistencies explicitly preserved and flagged within an isolated notes field. This strict architectural rigidity ensures that mathematical impossibilities are permanently logged, rendering the ledger a mathematically verifiable testament to the limits of discrete topographical parameterization.

3. Data Ledger

The following JSON array and accompanying summary table represent the unbroken cryptographic sequence of geometric derivations, preserving the strictly enforced dataset through sequential block validation.

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    "notes": "",
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      "dt_Gyr": 0.025
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    "notes": "",
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    "formula": "delta = length_B - C_new",
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      "delta_pct": 12.32
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    "notes": "FLAGGED: exceeds max achievable circle (great circle) on this sphere. Not
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      "C_new_Gly": 292.69,
      "R_new_Gly": 46.583
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    "formula": "phi = arccos(length_A / C_new); d = R_new*sin(phi)",
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      "d_Gly": 2.78
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    "notes": "Exact small-circle offset reproducing length A on the grown sphere",
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```

```
    "output": {
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      "constraint": "absolute, not a design choice"
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    "notes": "Length B (328.744 Gly) categorically exceeds this bound at any theta; flagged, not corrected",
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    "this_block_hash": "ce2e52b36d3a70b5b6a9c4fe2e02f2ff102f03e8d36a5fa0c41cc9c40c1f8d57"
  }
]
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Block	Step Label	Inputs	Formula	Output
1	radius_future	R_{today}, H_0	Verbatim Dataset Assumption	52.325 Gly
2	circumference_inner	R_{today}	2\pi \cdot R_{today}	292.168 Gly
3	circumference_outer	R_{future}	2\pi \cdot R_{future}	328.744 Gly
4	annulus_area	R_{today}, R_{future}	\pi(R_{future}^2 - R_{today}^2)	1808.3 Gly^2
5	extruded_volume	Annulus Area	Area \cdot (dR / 3)	3511.0 Gly^3
6	sphere_grown_25 Myr	R_{today}	Temporal interpolation	R_{new} = 46.583 Gly; C_{new} = 292.69 Gly
7	length_A_comparison	C_{new}	Length A evaluation	Short by 0.518 Gly (0.18%)
8	length_B_comparison	C_{new}	Length B evaluation	Exceeds by 36.06 Gly (12.3%)
9	latitude_offset_A	N/A	\arccos((C_{new} - 0.518) / C_{new})	\phi \approx 3.43^\circ, d \approx 2.78 Gly
10	governing_bound_declaration	N/A	C(\theta) = 2\pi R\cdot \sin(\theta) \le 2\pi R	Strict Manifold Boundary Enforced

4. Results

Parameter	Value
R_{today}	46.5 Gly
H_0	0.0716 Gyr^{-1}
R_{future} (+1750 Myr)	52.325 Gly
C_{inner}	292.168 Gly
C_{outer}	328.744 Gly

Parameter	Value
Annulus area	1808.3 Gly^2
Extruded volume	3511.0 Gly^3
R_{new} (+25 Myr)	46.583 Gly
C_{new}	292.69 Gly
Length A vs C_{new}	Short by 0.518 Gly (0.18%)
Length B vs C_{new}	Exceeds by 36.06 Gly (12.3%)
Latitude offset	\phi \approx 3.43^\circ, d \approx 2.78 Gly
Governing bound	C(\theta) = 2\pi R\cdot \sin\theta \le 2\pi R

5. Limitations

The robust cryptographic chaining of the expanding geometric ledger is subject to rigid, fundamental mathematical limitations that govern both the absolute physical bounds of flat-space intersections and the severe pathological divergence inherent in approximating smooth curved surfaces through discrete computational boundaries. These limitations invalidate arbitrary spatial scaling and restrict the topological assertions that can be logically drawn from the data set.

The Absolute Geometric Constraint of Latitudinal Bounds

The spatial derivations mapping planar intersections against the expanding cosmic sphere dictate an unyielding bounding limit defined strictly by the equation C(\theta) = 2\pi R \cdot \sin\theta \le 2\pi R. It is imperative to state that this \sin\theta \le 1 boundary must be understood as an absolute mathematical and physical constraint governing spatial manifolds, not merely a conditional design choice of the ledger format.

In a perfectly Euclidean framework (\Omega_k = 0), calculating the one-dimensional perimeter of any two-dimensional planar slice traversing a bounded three-dimensional sphere is intrinsically limited by the sphere's maximum cross-section. The maximum circumference is generated exclusively at the equatorial plane, representing a great circle. Any rotation, translation, or polar displacement of this traversing plane away from the exact center of the manifold generates a small circle, where the polar angle \theta (the colatitude, defined as \frac{\pi}{2} - \phi, with \phi representing latitude) dictates the dimensional scaling. The multiplicative coefficient supplied by the sine function restricts the circumference to a maximum scalar of 1.0, achieved strictly when \theta = \frac{\pi}{2} radians (90^\circ).

This mathematical absolute perfectly frames the divergent properties of the two length measurements tested at the +25 Myr intermediary boundary (R_{new} = 46.583 Gly). The equatorial maximum circumference parameter is rigorously calculated as C_{new} = 292.69 Gly. When Length A exhibits a deficit of 0.518 Gly, rendering it 0.18\% shorter than C_{new}, it conforms entirely to the continuous spatial geometry of the sphere. This precise reduction represents a planar intersection displaced off the great circle. Specifically, the latitudinal offset is verified mathematically via the arc cosine of the fractional reduction, establishing a latitudinal displacement of \phi \approx 3.43^\circ. Projecting this angular shift along the curved surface of the manifold yields an exact arc length traversal d \approx 2.78 Gly from the equator. The ledger flawlessly contextualizes Length A as a geometrically valid small circle boundary.

Conversely, Length B attempts to enforce a circumference of +36.06 Gly beyond the exact boundary of C_{new}, manifesting an expansive deviation of 12.3\%. Because the bound C(\theta) \le 2\pi R dictates the absolute maximum for any linear, flat-plane intersection, Length

B violates the physical constraints of the three-dimensional manifold. For a closed, continuous curve to exceed 2\pi R on a bounded sphere, the curve must definitively abandon the two-dimensional planar geometry required by the ledger interface. Such a length forces the curve into a highly convoluted, non-planar topological state, oscillating wildly across multiple latitudinal boundaries. Because the overarching structural intent of the ledger is the mapping of "Trapezium-Spherical Two-Dimensional" intersections, the accommodation of Length B within a flat plane constitutes an absolute topological transgression, forbidden mathematically by the bounded sine function.

The Schwarz Lantern Caveat and Pathological Divergences in Surface Approximations

While the ledger successfully governs one-dimensional curvilinear boundaries such as circumferences through the C(\theta) constraint, the extrapolation of these parameters into massive two-dimensional area computations and three-dimensional volumetric extrusions introduces an extraordinarily deep mathematical vulnerability. The calculations mapping the exact annulus area (1808.3 Gly^2) and the extruded volume (3511.0 Gly^3) intrinsically rely upon the assumption that a discretely calculated mathematical mesh can reliably approximate continuous spatial expansion over vast temporal spans. This assumption is heavily curtailed by the Schwarz lantern caveat, which strictly proves that the positional convergence of vertices on a curved surface does not guarantee the convergence of the derived surface area.
The Schwarz lantern, first documented by mathematician Hermann Amandus Schwarz in 1880, stands as a fundamental, pathological counterexample to naive topological notions regarding surface area limits. Originally designed to highlight the profound difficulty of defining the surface area of a smooth, curved manifold via the limiting process of inscribed polyhedra, the construction divides a simple cylinder of radius r and height h into discrete computational grids. Schwarz's theoretical model stacks M horizontal rings along the height of the cylinder, with each individual ring consisting of N equally spaced geometric vertices. To construct the discrete mesh approximating the continuous curved surface, consecutive rings are staggered, rotated relative to each other by an exact angle of \pi/N. This rotational stagger forms a continuous zigzag pattern of flat, isosceles triangles mapped tightly against the cylindrical wall.
Intuition, and indeed the historical definitions of surface area prior to Schwarz, assumed that geometric convergence was inevitable; if one infinitely increases the number of rings (M \to \infty) and simultaneously increases the number of vertices on each ring (N \to \infty), the discrete polyhedral mesh should perfectly mimic the smooth cylinder, with the area of the mesh converging securely to 2\pi rh. The Schwarz paradox entirely demolishes this assumption. Rigorous mathematical proofs governing the total surface area of this discrete triangulation reveal that mere proximity of the sampled coordinate points to the physical boundary is grossly inadequate. When comparing the total area of the flat, inscribed triangulation (\bar{S}) against the exact smooth surface area of a continuous cylindrical boundary (S), the fractional error is heavily bound by a complex interplay of polynomial ratios dictated entirely by M and N. The area of a singular flat triangle within this discrete grid, \bar{A}_{pqr}, inherently possesses an error margin relative to the true curved patch it attempts to cover, A_{pqr}. When integrated across the complete surface, the discrepancy bounded by the error margin demonstrates that the limit is violently unstable:
\frac{4\pi^4}{3N^2} - \frac{4\pi^6}{45N^4}(2 + 45M^2) < S^2 - \bar{S}^2 < \frac{4\pi^4}{3N^2}
This governing inequality explicitly proves that an infinite expansion of sampling points (M and

N) does not automatically yield the correct area. For the fractional topological error to vanish completely, the system demands an incredibly specific, non-intuitive limit: the ratio \frac{M}{N^2} must strictly approach 0.
If the computational mesh attempts to sample the vertical rings (M) proportionally to or faster than the square of the vertices (N^2), the total area of the inscribed polyhedron diverges explosively toward infinity, despite every individual vertex mathematically touching the continuous curved cylinder. As the triangulation becomes exceedingly dense, the flat triangles fold sharply inward and outward resembling a corrugated paper lantern, causing their individual flat areas to inflate wildly out of proportion to the minimal surface they attempt to cover. The limit of the area of the polyhedral surface depends entirely on *how* the variables tend to infinity, resulting in what calculus defines conditionally—rather than absolutely—convergent frameworks. This reflects the deepest structural truths of Riemann's rearrangement theorem for conditionally convergent series, clarifying that the mathematical architecture of the asymptotic approach dictates the survival or destruction of the calculation.
Within the scope of the digital geometric ledger calculating the cosmos, the Schwarz lantern caveat introduces an unbreakable requirement: the absolute necessity of facet-normal convergence. The pathological divergence of area in Schwarz's model occurs because, as the triangles fold, their perpendicular normal vectors tilt sharply away from the true normal vector of the smooth cylinder. Therefore, pointwise and uniform geometric convergence of the mesh coordinates to the curved boundary fails unless the mathematical field guarantees that the discrete facet normals also align perfectly with the smooth spatial normal vectors.
This caveat must be explicitly applied to any claim regarding surface-level quantities—such as the massive 1808.3 Gly^2 annulus area and the subsequent 3511.0 Gly^3 extruded volume encoded into the ledger. While inscribed polygonal chains accurately and reliably approximate one-dimensional arc lengths (securing the legitimacy of the C_{inner} and C_{outer} dimensional metrics), extending simple curve-length logic into two-dimensional area extrapolations invites catastrophic failure.
The resolution of these divergences in modern computational geometry, particularly within finite element methods applied to physical and spatial simulation, demands strict constraints on the geometric shape of the triangulated computational cells. Seminal proofs established by Babuška and Aziz dictate that convergence in discrete modeling requires the maximum angle of any approximating triangle to be rigidly bounded away from 180^\circ (\pi radians). If the angles within the computational mesh grow arbitrarily flat, mimicking the severely compressed, folded state of the triangles in an improperly scaled Schwarz lantern, convergence is utterly spoiled and the spatial calculations fail.
Furthermore, advanced topographies mapping spatial surfaces often rely on Discrete Orthogonal Geodesic (DOG) nets, which represent developable shapes via complex quadrilateral meshes governed by uncompromising local angle constraints. The mathematical viability of these nets fundamentally requires adequate, corrected normal vector fields to circumvent the Schwarz paradox. Different sampling rates of curved analytical geometries produce drastically different cotangent Laplacian normal approximations, rendering intrinsic measurements mathematically useless unless orthogonal geodesic parameterization successfully maps the discrete flat cells against the continuous smooth curve.
Consequently, the digital blockchain ledger documented here relies on exact, idealized mathematical formulations derived algebraically rather than via triangulated approximations. The area and volume metrics are exact expressions derived from flat-space analytical continuous functions, explicitly to bypass the discrete summation failures proved by the Schwarz paradox. However, any subsequent operational attempt to render, grid, or digitally map

these massive cosmic boundary areas using planar computational subdivisions will immediately fall subject to the binding necessity of facet-normal convergence. Failing to account for the angular relation of the planar projection relative to the smooth FLRW boundary will result in area formulations diverging violently to infinity, completely unraveling the cryptographic integrity of the dimensional data chain.
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